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Statistical Design and Performance of
High-Sensitivity Frequency-Feedback Receivers

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31 MAY 1963

Prepared by JEAN A. DEVELET, JR.
Electronics Research Laboratory

Prepared for COMMANDER SPACE SYSTEMS DIVISION

UNITED STATES AIR FORCE

Inglewood, California

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STATISTICAL DESIGN AND PERFORMANCE OF
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Prepared

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↙
A design theory and an analytical threshold determination are presented for a high-sensitivity frequency-feedback receiver.

ABSTRACT

↙
Current interest in high-sensitivity receivers for frequency modulation is centered around two implementations of the device: phase-lock and frequency-feedback. Enloe and later Cahn presented linearized frequency-feedback receiver design theory based on a twin threshold concept.

↘
The derivation of a quasi-linear model for the frequency-feedback receiver, for the situation of Gaussian signals and noise, is presented. A statistical optimization is then performed giving the loop transfer function and threshold performance for a maximum sensitivity receiver.

The design theory is based on a single threshold concept discarding the previous twin threshold approach. It is shown that in an optimum design the modulation error is less than 1 rad. This allows the use of a narrow-band i-f filter and obviates the need to consider threshold characteristics of the discriminator within the loop.

A significant result of this analysis is that although the quasi-linear receiver models differ in detail, threshold for a maximum sensitivity FM feedback and phase-lock receiver is identical. It was found that in FM feedback, as previously determined by Develet for the phase-lock receiver, threshold occurs at $10 \log_{10} (e) = 4.34$ db above the ultimate limit determined by information theory arguments. This 4.34 db degradation is independent of modulation index.

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It is anticipated that the results in this paper will free the design engineer from concern about which device is theoretically better and allow his choice to be influenced solely by hardware considerations.

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I. INTRODUCTION

Previous analyses of FM feedback receiver performance were based on a twin threshold concept which considered both open- and closed-loop thresholds (Refs. 1, 2, 3).

The purpose of this report is to show that the consideration of an open-loop threshold to enhance sensitivity by allowing greater modulation errors is not necessary in an optimum design. In fact, Cahn (Ref. 3) has shown that a design approach influenced by open-loop threshold for FM feedback yields a receiver sensitivity inferior to the phase-lock loop at large modulation indices.

A design theory which does not depend on an open-loop threshold will be presented along with an analytical threshold determination. It is found that the performance of the resulting receiver is identical to the phase-lock loop and therefore superior in sensitivity to the twin threshold design.

The principal analytic tool used here is Booton's quasi-linearization technique (Ref. 4) applied successfully in phase-lock threshold analysis (Ref. 5).

As with the analysis in Ref. 5, only signals with Gaussian amplitude probability distributions will be considered. Extension to other situations is possible but not treated in this report. The disturbing channel noise will be assumed white and Gaussian.

II. ANALYSIS

Consider the FM feedback receiver of Fig. 1.

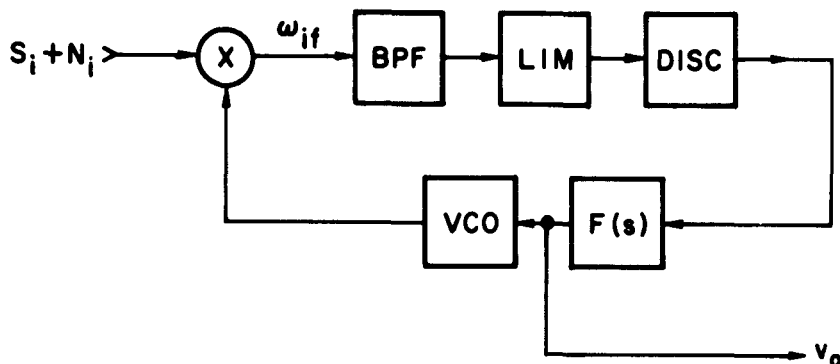


Fig. 1. FM Feedback Receiver

The received signal and noise are mixed to an i-f frequency, ω_{if} , by the local reference which attempts to follow the received signal faithfully in frequency. The band-pass filter (BPF) then removes noise from the compressed signal such that the standard discriminator will operate in a linear manner. The loop compensation network, $F(s)$, must be carefully chosen so that for the signals and noise under investigation a maximum sensitivity design results. The connection back to the voltage-controlled oscillator (VCO) completes the feedback path. The demodulated baseband output voltage is usually obtained from the VCO input and may be further filtered outside the loop to enhance output signal-to-noise ratio without affecting receiver sensitivity.

In the following analysis, the BPF will be assumed a very narrow-band simple-pole such as might be realized by a Q-multiplier. The noise bandwidth will be assumed small enough to insure linear operation of the frequency discriminator well above threshold.

Consider now the following input signal to the receiver of Fig. 1.

$$v_i(t) = E_i \sin [\omega_i t + \phi_i(t)] + N(t) \quad (1)$$

where

E_i = signal amplitude, v

ω_i = signal radian frequency, rad/sec

$\phi_i(t)$ = instantaneous signal modulation, rad

$N(t)$ = corrupting white Gaussian noise of
one-sided spectral density Φ , w/cps

Assume that $N(t)$ is of very broad spectral width compared to the signal but not so broad as to cause foldover in the subsequent mixing process. Let the VCO output voltage be given by

$$v_o(t) = 2 \cos [\omega_o t + \phi_o(t)] \quad (2)$$

where

ω_o = VCO radian frequency, rad/sec

$\phi_o(t)$ = VCO instantaneous phase, rad

Under these conditions, it may be shown that the input to the BPF in the vicinity of ω_{if} is given by

$$v_{if}(t) = E_i \sin [\omega_{if} t + \epsilon(t)] + x(t) \sin \omega_{if} t + y(t) \cos \omega_{if} t \quad (3)$$

where

ω_{if} = $\omega_i - \omega_o$, intermediate radian frequency, rad/sec

$\epsilon(t)$ = $\phi_i(t) - \phi_o(t)$, instantaneous loop error signal, rad

$x(t), y(t)$ = uncorrelated white Gaussian voltages of one-sided spectral density 2Φ , w/cps

Equation (3) may be rearranged as follows:

$$v_{if}(t) = [E_i \cos \epsilon(t) + x(t)] \sin \omega_{if} t + [E_i \sin \epsilon(t) + y(t)] \cos \omega_{if} t \quad (4)$$

The essence of the receiver analysis is the determination of the output of the narrow-band BPF when driven by the voltage of Eq. (4). In Fig. 2, the operation of the filter on Eq. (4) is depicted vectorially.

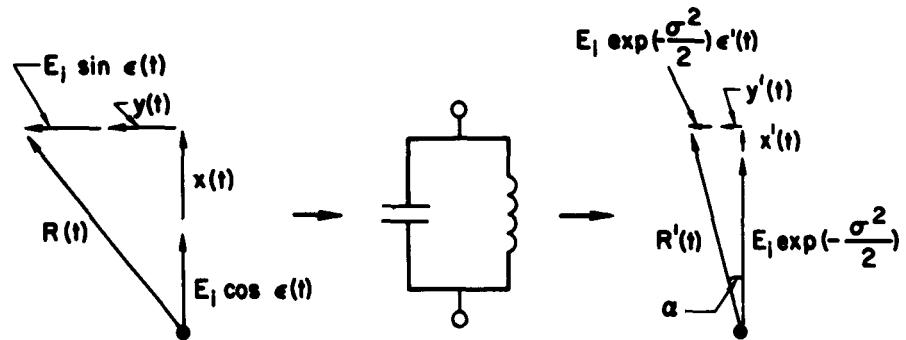


Fig. 2. Transfer Function of the Narrow-Band Filter

Assuming a narrow-band circuit of 3 db bandwidth, ω_0 rad/sec, it is clear that the additive noise variables x and y become new Gaussian variables x' and y' with a one-sided spectral density of a simple-pole low-pass nature and 3 db bandwidth $\omega_0/2$ rad/sec.

The signal terms $E_i \cos \epsilon(t)$ and $E_i \sin \epsilon(t)$ require closer examination.

$E_i \cos \epsilon(t)$ contains the signal carrier with fluctuations imposed by $\epsilon(t)$. A very narrow BPF compared to the fluctuation rate of $\epsilon(t)$ will yield for this component an output equal to $E_i \overline{\cos \epsilon(t)}$ with small variation about this value. Since these small variations are on the in phase vector (see Fig. 2), the angle α is little influenced by them. Assuming that $\epsilon(t)$ is Gaussian,¹ evaluation of the carrier output yields (see Ref. 5, Eq. 4):

$$E_i \overline{\cos \epsilon(t)} = E_i \exp\left(-\frac{\sigma^2}{2}\right) \quad (5)$$

where $\sigma^2 = \overline{\epsilon(t)^2}$, rad².

The $E_i \sin \epsilon(t)$ term contains the loop control function with the necessary information to properly drive subsequent circuits. After Booton (see Ref. 4, p. 372), this term will be approximated by an equivalent linear gain $K_{eq}(\sigma)$, whose value for any particular σ minimizes the mean-square difference from the actual nonlinear $E_i \sin [\]$ function (see Ref. 5, Eq. 4). Thus

$$E_i \sin \epsilon(t) \cong K_{eq}(\sigma) \epsilon(t) = E_i \exp\left(-\frac{\sigma^2}{2}\right) \epsilon(t) \quad (6)$$

¹It is characteristic of Booton's quasi-linearization to consider the statistics of $\epsilon(t)$ to be unaltered by system nonlinearities. See Ref. 4, p. 375.

Upon passage through the narrow-band filter, the spectral density of $\epsilon(t)$ will be altered, as in the case of the noise, by a simple-pole low-pass filter function. The quadrature signal output finally becomes

$$E_i \exp\left(-\frac{\sigma^2}{2}\right) \epsilon'(t) \quad (7)$$

Since a narrow-band filter was assumed, x' , y' , and $E_i \exp(-\sigma^2/2) \epsilon'(t)$ are all small compared to the carrier $E_i \exp(-\sigma^2/2)$, which is not attenuated at all by the BPF. The discriminator output is given therefore by (Fig. 2):

$$\dot{a} = \epsilon'(t) + \frac{\dot{y}'}{E_i \exp(-\sigma^2/2)} \quad (8)$$

Equation (8) is a key relation from which the quasi-linear model of the FM feedback receiver may be constructed. In Fig. (3), the quasi-linear FM

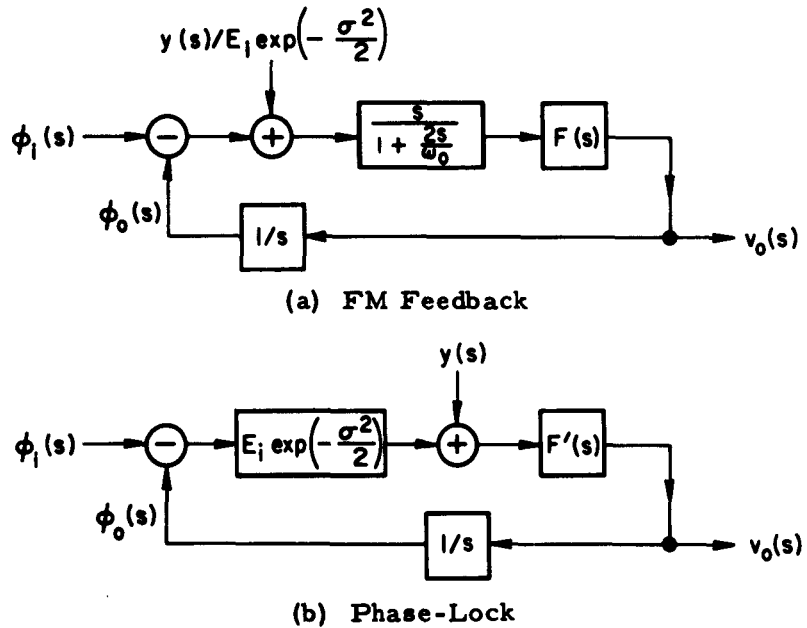


Fig. 3. Quasi-Linear Receiver Models

feedback receiver model and the quasi-linear phase-lock receiver model (see Ref. 5, Fig. 3) are depicted. The transfer function, $s/(1 + 2s/\omega_o)$, results from the combined action of the discriminator and BPF.

It is interesting to compare these models for the two implementations. Note that either a decrease in signal strength or an increase in loop error tends to enhance the equivalent noise input in FM feedback, while the loop parameters remain fixed. This is a direct result of the limiter which precedes discrimination in the frequency-feedback demodulator of Fig. 1. On the other hand, in the phase-lock receiver the input noise remains fixed, but the loop gain changes with either signal strength or loop error.

This latter characteristic of phase-lock receivers has long been known. Phase-lock loops are usually optimally designed for a particular set of conditions, e.g., noise, loop error, and signal strength at threshold. Parameter variation at other conditions is accepted as characteristic of the device (Ref. 6). The tendency for parameters to remain fixed in the FM feedback receiver of Fig. 1 may be advantageous in certain situations.

The detailed differences of the two models given in Fig. 3 in no way influence the ultimate sensitivity of an optimally designed receiver. Configurations 3(a) and 3(b) are equivalent servomechanisms if $F(s)$ and $F'(s)$ are selected to make ϕ_o/ϕ_i identical in both cases.

Let us return now to the FM feedback receiver analysis. As a result of the $\exp(-\sigma^2/2)$ in the noise term, the quasi-linear model yields a precise threshold criterion. In order to determine this threshold, the closed-loop transfer function is defined as

$$\frac{\phi_o}{\phi_i}(s) = \frac{F(s)/(1 + 2s/\omega_o)}{1 + F(s)/(1 + 2s/\omega_o)} \quad (9)$$

It will be assumed that $F(s)$ contains any loop gain functions, e.g., VCO, discriminator, and amplifiers.

The mean-square value of loop error, σ^2 , may now be written as

$$\sigma^2 = \int_0^\infty \Phi_m(\omega) \left| 1 - \frac{\phi_o}{\phi_i} \right|^2 df + \int_0^\infty \frac{\Phi \exp(\sigma^2)}{S} \left| \frac{\phi_o}{\phi_i} \right|^2 df \quad (10)$$

where

$\Phi_m(\omega)$ = one-sided spectral density of $\phi_i(t)$, rad^2/cps

S = input signal power $E_i^2/2$, w

Equation (10) is a familiar result and is identical to the total loop error for the quasi-linear phase-lock receiver. It has been shown that minimization of Eq. (10) by proper choice of ϕ_o/ϕ_i also minimizes the required received signal power, S , and thereby yields a design of maximum sensitivity (see Ref. 5, p. 351).

It can be concluded, therefore, that within the approximation of quasi-linearization the performance of an FM feedback receiver with a narrow-band i-f filter is identical to a phase-lock receiver when each has the same closed-loop transfer function ϕ_o/ϕ_i .²

The threshold determined by Eq. (10) has been thoroughly discussed in a previous paper (see Ref. 5). Figure 4 (from Ref. 5, Fig. 5) depicts the case where $\Phi_m(\omega)$ is a constant Φ_m for $0 < f < f_m$ and zero elsewhere.

As with the phase-lock receiver, it is seen that the optimal FM feedback receiver performance is 4.34 db poorer than Shannon's limit at all modulation indices. The second-order FM feedback receiver is 2 - 3 db poorer than the optimal receiver in the region of normally encountered output signal-to-noise power ratios.

²As noted previously, $F(s)$ will differ in the two cases.

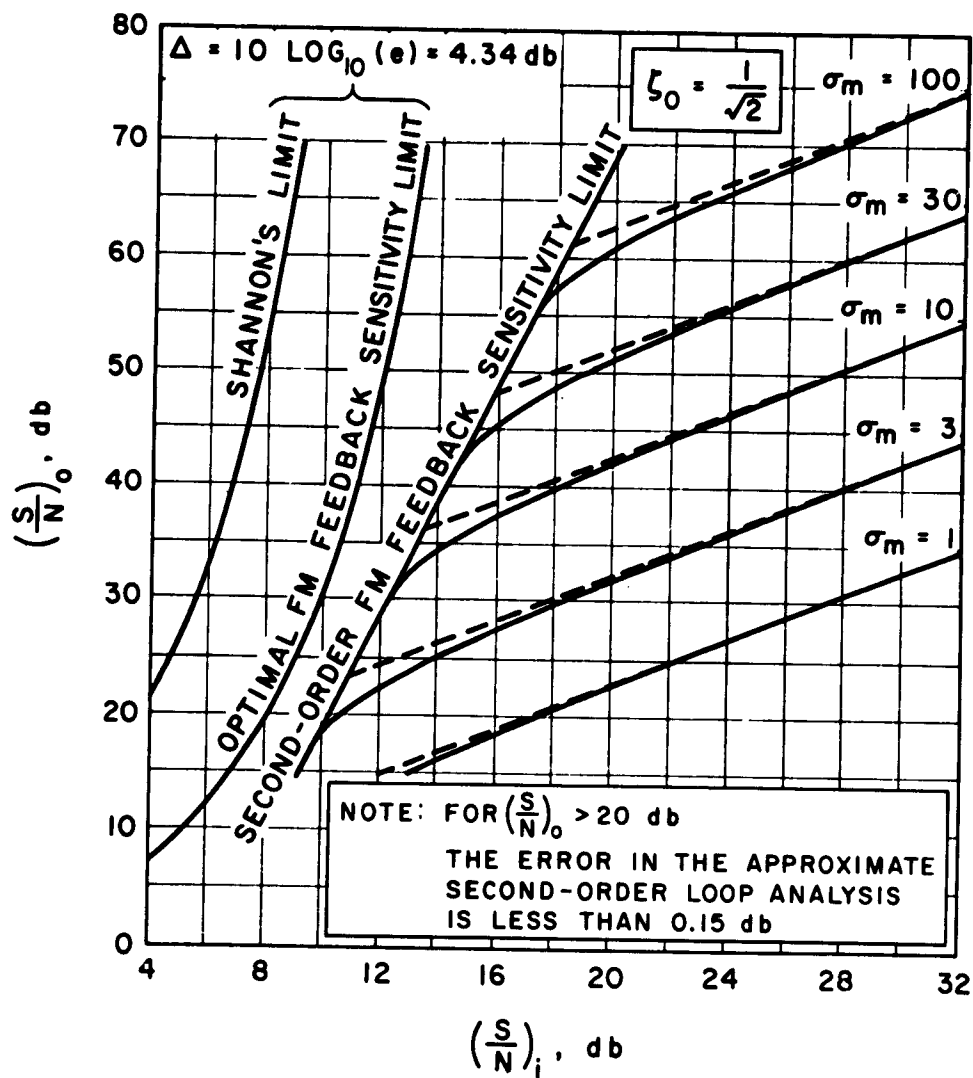


Fig. 4. Quasi-Linear Receiver Performance for the Situation of Bandlimited White Gaussian Phase-Encoded Signals

The definitions of the various terms in Fig. 4 are repeated below for completeness.

$$\left(\frac{S}{N}\right)_i = \frac{S}{2\Phi f_m} \quad (11)$$

$$\sigma_m = \sqrt{\Phi f_m} \quad (12)$$

$$\left(\frac{S}{N}\right)_o = \frac{2\sigma_m^2 (S/N)_i}{\exp(\sigma^2)} \quad (13)$$

The realizable transfer function which maximizes receiver sensitivity for arbitrary signal spectra in white noise is given by (see Ref. 5, Eq. 13):

$$\left|1 - \frac{\phi_o}{\phi_i}\right|^2 = \frac{\Phi \exp(\sigma^2)/S}{\Phi \exp(\sigma^2)/S + \Phi_m(\omega)} \quad (14)$$

Bode's minimum phase relation in conjunction with Eq. (14) may be used to find ϕ_o/ϕ_i (Ref. 7).

The total loop error, σ , and the threshold relation for the situation of the optimum realizable transfer function are obtained from (see Ref. 5, Eq. 14):

$$\sigma^2 = \frac{\Phi \exp(\sigma^2)}{S} \int_0^\infty \log_e \left[1 + \frac{\Phi_m(\omega)S}{\Phi \exp(\sigma^2)} \right] df \quad (15)$$

It can be questioned whether or not threshold could be further enhanced by increasing the bandwidth of the i-f filter which would allow for greater modulation error. For nonoptimum loop design, threshold is enhanced. However, if one strives for an optimum design (transfer function given by Eq. 14), the threshold value for the loop error, σ , invariably lies in the vicinity of 1 rad.³ Since modulation error is only a small fraction of the total error, σ , especially at high indices,⁴ it is clear that little is to be gained by special accommodations for a negligible quantity. In nonoptimum designs, however, the increased modulation error which results will require a wider i-f filter to achieve maximum sensitivity.

Actual side-by-side comparisons are in order for conclusive proof of the advantage of nonoptimum FM feedback design versus nonoptimum phase-lock receiver design. A treatment of this subject is beyond the scope of this report.

³Identically 1 rad for $\Phi_m(\omega) = \Phi_m$ for $0 < f < f_m$.

⁴It is characteristic of Wiener-Hopf solutions for the signal following error to decrease at high signal-to-noise ratios (large modulation indices). For proof of this fact for the modulation form of Fig. 4 see J. A. Develet, Jr., "Coherent FDM/FM Telephone Communication," Proc. IRE, September 1962, footnote p. 1960.

III. CONCLUSIONS

A simple receiver model and a threshold criterion for the FM feedback receiver have been developed. Only the situation of Gaussian signals and noise was considered. Considering that the modulation error becomes vanishingly small at high indices, a generalization to arbitrary signal probability distributions may be valid as long as the noise remains Gaussian.

The principal analytical tool was the quasi-linearization technique developed by Booton for analysis of nonlinear servosystems with statistical inputs.

A significant tactic in the analysis was the introduction of the narrow-band i-f filter to eliminate irrelevant open-loop threshold considerations. This approach is in direct contradiction to all previously suggested design techniques (Refs. 1, 2, 3). These previous techniques must consider open-loop threshold, since the Bode filter (Refs. 1, 2) and a constrained compression optimization (Ref. 3) do not collapse modulation error to zero at high indices as does the design procedure suggested here. Cahn (Ref. 3) demonstrated that an FM feedback receiver influenced by open-loop threshold becomes inferior to the phase-lock loop at large modulation indices. On the other hand, by following the design procedure set forth in this paper, it was shown that the FM feedback receiver in its most sensitive form performs identically to the phase-lock receiver. The ultimate sensitivity is within 4.34 db of the theoretical limit. This 4.34 db degradation holds regardless of modulation index.

Finally, it is observed, as first noted by Chaffee (Ref. 8) in the original work on this device, that the limiter in Fig. 1 is unessential. Postulating a very narrow i-f filter, the fluctuations in the amplitude of the filter response are negligible (see Fig. 2) and require no limiting to reduce their effect on the discriminator output. Of course, without the limiter, parameter variation will occur with signal strength and loop error as in the phase-lock loop.

It is hoped that the results presented herein will allow a choice between either the FM feedback or the phase-lock approach to receiver design to be influenced by hardware complexity rather than theoretical performance.

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